What Reichenbach Proved about Common Causes

Elliott Sober

In Reichenbach's (1956, p. 157) example of the two actors in the traveling acting troupe, each actor has a probability of getting sick on a day if he/she eats tainted food on that day, and each has a probability of getting sick on a day if he/she does not eat tainted food on that day. Let "A_1" represent the proposition that the first actor gets sick, and let "A_2" represent the proposition that the second actor gets sick. The common cause hypothesis says that the actors take their meals together, each day in a different restaurant. So let "R=t" mean that the restaurant at which they both dine serves tainted food, and let "R=n" mean that the restaurant at which they both dine serves nontainted food.

Here is some notation for the probabilities just described:

$Pr(A_1 | R=t) = x$ $\quad$ $Pr(A_1 | R=n) = a$

$Pr(A_2 | R=t) = y$ $\quad$ $Pr(A_2 | R=n) = b$

Question: must the two items in the first line sum to one? The two in the second?

Finally, let $Pr(R=t) = e$. [So $Pr(R=n) =$ ____]

It follows that

$$Pr(A_1) = xe + a(1-e)$$

$$Pr(A_2) = ye + b(1-e)$$

Assume that the state of the restaurant screens-off A_1 from A_2 (that is, that A_1 and A_2 are probabilistically independent when you conditionalize on R=t, and also when you conditionalize on R=n). This means that

$$Pr(A_1 \& A_2 | R=t) = Pr(A_1 | R=t)Pr(A_2 | R=t) = xy$$

$$Pr(A_1 \& A_2 | R=n) = Pr(A_1 | R=n)Pr(A_2 | R=n) = ab$$

This entails that

$$Pr(A_1 \& A_2) = xye + ab(1-e).$$

Your assignment: (1) Prove from the above that A_1 and A_2 will be positively correlated. Hint: you may need a few assumptions additional to the ones listed above to derive this result. If so, state what those assumptions are. Second hint: start by writing out what the claim of positive correlation asserts. (2) Reichenbach is famous for stating a "principle of the common cause," one version of which says: If A_1 and A_2 are correlated, then they have a common cause. Does this principle follow from what Reichenbach proved about the acting example?

Reference: Hans Reichenbach, *The Direction of Time*, University of California Press.