

**When is altruism fitter than selfishness?
A simple example in which groups are of size 2**

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Consider a population composed of numerous pairs of individuals. The individuals in a pair interact with each other in ways that affect each other's fitnesses. The following table describes how your fitness depends on your own phenotype and on the phenotype of your partner.

		Your partner is	
		Altruistic	Selfish
You are	Altruistic	$x+b-c$	$x-c$
	Selfish	$x+b$	x

If you are an altruist, you pay a cost c ; if you are selfish, you pay nothing. If your partner is an altruist, you receive a benefit b ; if your partner is selfish, you receive nothing. Altruists will have different fitnesses, depending on who their partners are; selfish individuals will also differ in fitness, for the same reason. Despite this variability, we can write out equations that describe the average fitnesses of altruism and selfishness in this population.

If you are an altruist, you have a probability p that your partner is an altruist and a probability $(1-p)$ that your partner is selfish. That is, there are two conditional probabilities:

$$p = \text{Pr}(\text{partner is an altruist} \mid \text{you are an altruist}); \quad (1-p) = \text{Pr}(\text{partner is selfish} \mid \text{you are an altruist})$$

This means that your fitness, if you are an altruist, is

$$\text{Fitness of } A = p(x-c+b) + (1-p)(x-c).$$

Similarly, if you are selfish, you have a probability q that your partner is altruistic and a probability $(1-q)$ that your partner is selfish. Here again q and $(1-q)$ are conditional probabilities that sum to 1:

$$q = \text{Pr}(\text{partner is an altruist} \mid \text{you are an selfish}); \quad (1-q) = \text{Pr}(\text{partner is selfish} \mid \text{you are an selfish})$$

So your fitness, if you are selfish, is

$$\text{Fitness of } S = q(x+b) + (1-q)(x).$$

It follows that

$$A \text{ is fitter than } S \text{ iff } p(x-c+b) + (1-p)(x-c) > q(x+b) + (1-q)x.$$

This simplifies to

$$A \text{ is fitter than } S \text{ iff } (p-q) > c/b.$$

Recalling what p and q mean (they are conditional probabilities), our result is that

$$(*) \quad A \text{ is fitter than } S \text{ iff } \text{Pr}(\text{partner is } A \mid \text{you are } A) - \text{Pr}(\text{your partner is } A \mid \text{you are } S) > c/b.$$

Questions: (1) Consider the expression “ $\text{Pr}(\text{partner is } A \mid \text{you are } A) - \text{Pr}(\text{your partner is } A \mid \text{you are } S)$.” What is the biggest this quantity can be? The smallest? What does this difference in the two conditional probabilities represent (see page 81 of *Philosophy of Biology*)? (2) Can altruism be fitter than selfishness if pairs form at random? (3) Which trait will be fitter if like always pairs with like? (4) Which trait will be fitter if pairs are always mixed?